Z4

Philip Feinsilver

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In [1]: %run sympowers.py
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This provides macros for symmetric tensor powers.

SymPower(A,N): a matrix A and degree N
SymmTraces(A,n): matrix A and order of the series
PowerTraces(A,n): matrix A and order of the series
GAM(A,N): Lie map for matrix A in degree N
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In [5]:  F=Matrix(2,2,[0,1,1,1])
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In [7]: [SymPower(F,i) for i in range(5)]
```

Out[7]:

$$\left[\begin{bmatrix}1\end{bmatrix}, \quad \begin{bmatrix}0 & 1\\1 & 1\end{bmatrix}, \quad \begin{bmatrix}0 & 0 & 1\\0 & 1 & 1\\1 & 2 & 1\end{bmatrix}, \quad \begin{bmatrix}0 & 0 & 0 & 1\\0 & 0 & 1 & 1\\0 & 1 & 2 & 1\\1 & 3 & 3 & 1\end{bmatrix}, \quad \begin{bmatrix}0 & 0 & 0 & 0 & 1\\0 & 0 & 0 & 1 & 1\\0 & 0 & 1 & 2 & 1\\0 & 1 & 3 & 3 & 1\\1 & 4 & 6 & 4 & 1\end{bmatrix}\right]$$

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In [9]: ee=list(SymPower(F,4).eigenvals())
        [nsimplify(ee[i],[GoldenRatio]) for i in range(5)]
```

Out[9]:

$$[1, \quad 2+3\phi, \quad -2+\phi, \quad -\phi-1, \quad -3\phi+5]$$

```
In [11]: phi=GoldenRatio
         [(i,nsimplify(phi**i,[GoldenRatio])) for i in range(10)]
```

Out[11]:

$$[(0, \quad 1), \quad (1, \quad \phi), \quad (2, \quad 1+\phi), \quad (3, \quad 1+2\phi), \quad (4, \quad 2+3\phi), \quad (5, \quad 3+5\phi), \quad (6, \quad 5+8\phi), \quad (7, \quad 8+$$

```
In [12]: [(i,nsimplify(phi**(-i),[GoldenRatio])) for i in range(10)]
```

Out[12]:

$$[(0,\ 1),\ (1,\ -1+\phi),\ (2,\ -\phi+2),\ (3,\ -3+2\phi),\ (4,\ -3\phi+5),\ (5,\ -8+5\phi),\ (6,\ -8\phi+1$$

```
In [13]:  R=Matrix(2,2,[1,0,1,1])
         [SymPower(R,i) for i in range(5)]
```

Out[13]:

$$\left[\begin{bmatrix}1\end{bmatrix},\ \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix},\ \begin{bmatrix}1 & 0 & 0\\1 & 1 & 0\\1 & 2 & 1\end{bmatrix},\ \begin{bmatrix}1 & 0 & 0 & 0\\1 & 1 & 0 & 0\\1 & 2 & 1 & 0\\1 & 3 & 3 & 1\end{bmatrix},\ \begin{bmatrix}1 & 0 & 0 & 0 & 0\\1 & 1 & 0 & 0 & 0\\1 & 2 & 1 & 0 & 0\\1 & 3 & 3 & 1 & 0\\1 & 4 & 6 & 4 & 1\end{bmatrix}\right]$$

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In [14]: D=Matrix(2,2,[phi,0,0,-phi**(-1)])
         D
```

Out[14]:

$$\begin{bmatrix}\phi & 0\\0 & -\frac{1}{\phi}\end{bmatrix}$$

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In [15]: [SymPower(D,i) for i in range(5)]
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Out[15]:

$$\left[\begin{bmatrix}1\end{bmatrix},\ \begin{bmatrix}\phi & 0\\0 & -\frac{1}{\phi}\end{bmatrix},\ \begin{bmatrix}\phi^2 & 0 & 0\\0 & -1 & 0\\0 & 0 & \frac{1}{\phi^2}\end{bmatrix},\ \begin{bmatrix}\phi^3 & 0 & 0 & 0\\0 & -\phi & 0 & 0\\0 & 0 & \frac{1}{\phi} & 0\\0 & 0 & 0 & -\frac{1}{\phi^3}\end{bmatrix},\ \begin{bmatrix}\phi^4 & 0 & 0 & 0 & 0\\0 & -\phi^2 & 0 & 0 & 0\\0 & 0 & 1 & 0 & 0\\0 & 0 & 0 & -\frac{1}{\phi^2} & 0\\0 & 0 & 0 & 0 & \frac{1}{\phi^4}\end{bmatrix}\right]$$

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In [20]: GF=GAM(F,3)
         ee=list(GF.eigenvals())
         GF,ee,[nsimplify(ee[i],[GoldenRatio]) for i in range(4)]
```

Out[20]:

$$\left(\begin{bmatrix}0 & 3 & 0 & 0\\1 & 1 & 2 & 0\\0 & 2 & 2 & 1\\0 & 0 & 3 & 3\end{bmatrix},\ \left[\frac{3}{2}+\frac{3\sqrt{5}}{2},\ -\frac{\sqrt{5}}{2}+\frac{3}{2},\ -\frac{3\sqrt{5}}{2}+\frac{3}{2},\ \frac{\sqrt{5}}{2}+\frac{3}{2}\right],\ [3\phi,\ -\phi+2,\ -3\phi+3,\ 1+\phi]\right)$$

$(-1)^k\,\phi^{N-2k}$

$(N-2k)\phi+k$

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In [ ]:
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