Symmetric Powers

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In [3]:
```
from IPython.display import *
```

In [4]:
```
var('a:z')

V=Matrix(3,1,[u,v,w])
A=Matrix(3,3,[s,1,1,1,0,0,0,1,0])
B=A*V
A,B
```

Out[4]:

$$\left(\begin{bmatrix} s & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} su+v+w \\ u \\ v \end{bmatrix} \right)$$

In [5]:
```
N=3
mm=Poly((u+v+w)**N).monoms()
mm
```

Out[5]:

[(3, 0, 0), (2, 1, 0), (2, 0, 1), (1, 2, 0), (1, 1, 1), (1, 0, 2), (0, 3, 0), (0, 2, 1), (0, 1, 2), (0, 0, 3)]

In [6]:
```
dd=len(mm)
xx=eye(dd)
X=[]

for i in range(dd):
    F=x**mm[i][0]*y**mm[i][1]*z**mm[i][2]
    GG=F.subs(x,B[0]).subs(y,B[1]).subs(z,B[2]);FF=expand(GG)
    X.append([FF.coeff(u**mm[i][0]*v**mm[i][1]*w**mm[i][2]) for i in range(dd)])

#display(X)
```

In [7]:
```
MX=Matrix(1,dd,X[0])
for i in range(1,dd):
    XM=Matrix(1,dd,X[i])
    MX=MX.col_join(XM)


MX
```

Out[7]:

$$\begin{bmatrix} s^3 & 3s^2 & 3s^2 & 3s & 6s & 3s & 1 & 3 & 3 & 1 \\ s^2 & 2s & 2s & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & s^2 & 0 & 2s & 2s & 0 & 1 & 2 & 1 & 0 \\ s & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & s & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

This is the Nth symmetric power of A. In this example, N=3.

In [8]:
```
IX=eye(A.shape[0])
delta=series(((IX-t*A).det())**(-1),t,0,10)
for i in range(10):
    display(delta.coeff(t,i))
```

$$1$$

$$s$$

$$s^2 + 1$$

$$s^3 + 2s + 1$$

$$s^4 + 3s^2 + 2s + 1$$

$$s^5 + 4s^3 + 3s^2 + 3s + 2$$

$$s^6 + 5s^4 + 4s^3 + 6s^2 + 6s + 2$$

$$s^7 + 6s^5 + 5s^4 + 10s^3 + 12s^2 + 7s + 3$$

$$s^8 + 7s^6 + 6s^5 + 15s^4 + 20s^3 + 16s^2 + 12s + 4$$

$$s^9 + 8s^7 + 7s^6 + 21s^5 + 30s^4 + 30s^3 + 30s^2 + 17s + 5$$

These are the traces of the symmetric powers of A.

In [9]:
```
Res=series(trace((IX-t*A).inv()),t,0,10)
for i in range(10):
    display(Res.coeff(t,i))
```

$$3$$

$$s$$

$$s^2+2$$

$$s^3+3s+3$$

$$s^4+4s^2+4s+2$$

$$s^5+5s^3+5s^2+5s+5$$

$$s^6+6s^4+6s^3+9s^2+12s+5$$

$$s^7+7s^5+7s^4+14s^3+21s^2+14s+7$$

$$s^8+8s^6+8s^5+20s^4+32s^3+28s^2+24s+10$$

$$s^9+9s^7+9s^6+27s^5+45s^4+48s^3+54s^2+36s+12$$

These are the traces of the powers of A.