# Symmetric Powers, Traces, and Lie Map

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In [44]: %run sympowers.py
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This provides macros for symmetric tensor powers.

SymPower(A,N): a matrix A and degree N
SymmTraces(A,n): matrix A and order of the series
PowerTraces(A,n): matrix A and order of the series
GAM(A,N): Lie map for matrix A in degree N
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In [45]: A=Matrix(2,2,[1,2,x,y])
         A
```

Out[45]:

$$\begin{bmatrix} 1 & 2 \\ x & y \end{bmatrix}$$

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In [46]: for n in range(1,6):
             B=SymPower(A,n)
             display(B,trace(B),expand(trace(A**n)))
```

$$\begin{bmatrix} 1 & 2 \\ x & y \end{bmatrix}$$

$$y + 1$$

$$y + 1$$

$$\begin{bmatrix} 1 & 4 & 4 \\ x & 2x + y & 2y \\ x^2 & 2xy & y^2 \end{bmatrix}$$

$$2x + y^2 + y + 1$$

$$4x + y^2 + 1$$

$$\begin{bmatrix} 1 & 6 & 12 & 8 \\ x & 4x + y & 4x + 4y & 4y \\ x^2 & 2x^2 + 2xy & 4xy + y^2 & 2y^2 \\ x^3 & 3x^2y & 3xy^2 & y^3 \end{bmatrix}$$

$$4xy + 4x + y^3 + y^2 + y + 1$$

$$6xy + 6x + y^3 + 1$$

$$\begin{bmatrix} 1 & 8 & 24 & 32 & 16 \\ x & 6x + y & 12x + 6y & 8x + 12y & 8y \\ x^2 & 4x^2 + 2xy & 4x^2 + 8xy + y^2 & 8xy + 4y^2 & 4y^2 \\ x^3 & 2x^3 + 3x^2y & 6x^2y + 3xy^2 & 6xy^2 + y^3 & 2y^3 \\ x^4 & 4x^3y & 6x^2y^2 & 4xy^3 & y^4 \end{bmatrix}$$

$$4x^2 + 6xy^2 + 8xy + 6x + y^4 + y^3 + y^2 + y + 1$$

$$8x^2 + 8xy^2 + 8xy + 8x + y^4 + 1$$

$$\begin{bmatrix} 1 & 10 & 40 & 80 & 80 & 32 \\ x & 8x + y & 24x + 8y & 32x + 24y & 16x + 32y & 16y \\ x^2 & 6x^2 + 2xy & 12x^2 + 12xy + y^2 & 8x^2 + 24xy + 6y^2 & 16xy + 12y^2 & 8y^2 \\ x^3 & 4x^3 + 3x^2y & 4x^3 + 12x^2y + 3xy^2 & 12x^2y + 12xy^2 + y^3 & 12xy^2 + 4y^3 & 4y^3 \\ x^4 & 2x^4 + 4x^3y & 8x^3y + 6x^2y^2 & 12x^2y^2 + 4xy^3 & 8xy^3 + y^4 & 2y^4 \\ x^5 & 5x^4y & 10x^3y^2 & 10x^2y^3 & 5xy^4 & y^5 \end{bmatrix}$$

$$12x^2y + 12x^2 + 8xy^3 + 12xy^2 + 12xy + 8x + y^5 + y^4 + y^3 + y^2 + y + 1$$

$$20x^2y + 20x^2 + 10xy^3 + 10xy^2 + 10xy + 10x + y^5 + 1$$

In [47]:
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st=SymmTraces(A,6);pt=PowerTraces(A,6)

for i in range(6):
    display([st[i],pt[i]])
```

$$[1, \quad 2]$$

$$[y + 1, \quad y + 1]$$

$$\left[2x + y^2 + y + 1, \quad 4x + y^2 + 1\right]$$

$$\left[4xy + 4x + y^3 + y^2 + y + 1, \quad 6xy + 6x + y^3 + 1\right]$$

$$\left[4x^2 + 6xy^2 + 8xy + 6x + y^4 + y^3 + y^2 + y + 1, \quad 8x^2 + 8xy^2 + 8xy + 8x + y^4 + 1\right]$$

$$\left[12x^2y + 12x^2 + 8xy^3 + 12xy^2 + 12xy + 8x + y^5 + y^4 + y^3 + y^2 + y + 1, \quad 20x^2y + 20x^2 + 10xy^3 + 10xy^2 + 10xy + 10x + y^5 + 1\right]$$

In [49]:
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B=Matrix(2,2,[y,x,2,1])
C=A*B-B*A
A,B,C
```

Out[49]:

$$\left(\begin{bmatrix}1 & 2\\ x & y\end{bmatrix}, \quad \begin{bmatrix}y & x\\ 2 & 1\end{bmatrix}, \quad \begin{bmatrix}-x^2+4 & -xy+x-2y+2\\ xy-x+2y-2 & x^2-4\end{bmatrix}\right)$$

In [53]:
```
GAM(A,4),GAM(B,4),GAM(C,4),simplify(GAM(A,4)*GAM(B,4)-GAM(B,4)*GAM(A,4)-GAM(C,4))
```

Out[53]:

$$\left(\begin{bmatrix}4 & 8 & 0 & 0 & 0\\ x & y+3 & 6 & 0 & 0\\ 0 & 2x & 2y+2 & 4 & 0\\ 0 & 0 & 3x & 3y+1 & 2\\ 0 & 0 & 0 & 4x & 4y\end{bmatrix}, \quad \begin{bmatrix}4y & 4x & 0 & 0 & 0\\ 2 & 3y+1 & 3x & 0 & 0\\ 0 & 4 & 2y+2 & 2x & 0\\ 0 & 0 & 6 & y+3 & x\\ 0 & 0 & 0 & 8 & 4\end{bmatrix}, \quad \begin{bmatrix}-4x^2+16 & -4xy+4x-8y+8 & 0 & 0 & 0\\ xy-x+2y-2 & -2x^2+8 & -3xy+3x-6y+6 & 0 & 0\\ 0 & 2xy-2x+4y-4 & 0 & -2xy+2x-4y+4 & 0\\ 0 & 0 & 3xy-3x+6y-6 & 2x^2-8 & -xy+x-2y+2\\ 0 & 0 & 0 & 4xy-4x+8y-8 & 4x^2-16\end{bmatrix}, \quad \begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\right)$$

In [ ]: